$E_{6(6)}$ Exceptional Field Theory: Applications to Type IIB Supergravity on $AdS_5 \times S^5$

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Motivations

- Search for a unified theory: reconciliate matter and gravity at the quantum level.
- Important historical example: Kaluza-Klein theory. Electromagnetism and gravity unification by embedding in a five dimensional spacetime. Provide mechanism for dimensional reduction.
- Idea still influential today $\longrightarrow$ consistent Kaluza-Klein truncation. Powerful tool: any solution of the lower-dimensional theory can be uplifted to the higher-dimensional theory.

A toy model

Free massless scalar field in (D+1) dimensional spacetime satisfy

$$\hat{\Box}\,\hat{\Phi}(x,y) = 0$$

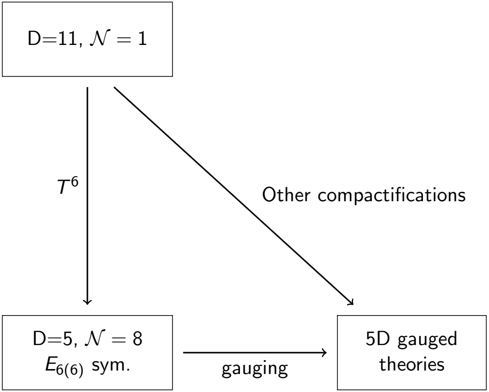
Now, suppose we compactify on circle of 'small' radius R

$$\hat{\Phi}(x,y) = \Sigma\phi_n(x)e^{iny/R} \;\rightarrow\; \Box\,\phi_n(x) - \frac{n^2}{R^2}\phi_n(x) = 0$$

In this case, truncating to the massless sector (n=0) is consistent.
General case could be more complicated, e.g. $\Box\phi_1 = \phi_0$, inconsistent.

Q: Are the fields I keep sources for the field I discard ?

Compactifications & symmetries



Dimensional reduction
→ enhanced symmetries.

Cremmer-Julia [1979]:
torus reduction of
D=11 SUGRA

$$D = 5 : E_{6(6)}$$
$$D = 4 : E_{7(7)}$$
$$D = 3 : E_{8(8)}$$

Q: Framework that make these exceptional symmetries manifest ?

The idea

Coordinate (x^μ, Y^M), $\mu = 0..4$, M: fundamental **27** of $E_{6(6)}$

$$
\begin{array}{rcl}
11 = 5 & + & 6 \\
 & & \downarrow \text{enlarged internal space} \\
5 & + & 27
\end{array}
$$

Then, get rid of any redundancy with the section constraint

$$d^{MNK}\partial_N\partial_K A = 0\,, \qquad d^{MNK}\partial_N A \partial_K B = 0$$

for any fields or gauge parameters A,B
and with d^{MNK} invariant tensor under **27**.

Content

Gauge parameter $\Lambda^M(x, Y)$: Local gauge transformations w.r.t to x-space.
$\rightarrow \partial_\mu$ need to be covariantized.

Introduce gauge connection $\mathcal{A}_\mu{}^M$ and define (external) covariant derivative

$$\mathcal{D}_\mu \equiv \partial_\mu - \mathbb{L}_{\mathcal{A}_\mu}, \quad \text{such that } \delta_\Lambda \mathcal{A}_\mu{}^M = \mathcal{D}_\mu \Lambda^M$$

Naive curvature: $F_{\mu\nu}{}^M = 2\partial_{[\mu}\mathcal{A}_{\nu]}{}^M - [A_\mu, A_\nu]_E{}^M \rightarrow$ not covariant.

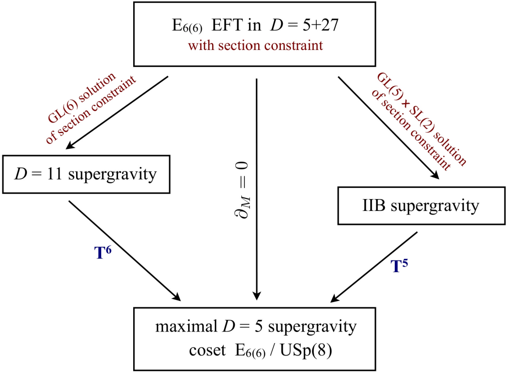
Tensor hierarchy: introduction of a two-form $\mathcal{B}_{\mu\nu\, M}$

$$\mathcal{F}_{\mu\nu}{}^M = F_{\mu\nu}{}^M + 10 d^{MNK} \partial_K \mathcal{B}_{\mu\nu\, N}$$

Satisfy generalized Bianchi identity

$$3\mathcal{D}_{[\mu}\mathcal{F}_{\nu\rho]}{}^M = 10 d^{MNK} \partial_K \mathcal{H}_{\mu\nu\rho\, N}$$

Solutions of the section constraints



$$\mathrm{E}_{6(6)} \longrightarrow \mathrm{GL}(5) \times \mathrm{SL}(2)$$

$$\mathbf{27} \longrightarrow (\mathbf{5},\mathbf{1}) \oplus (\mathbf{5}',\mathbf{2}) \oplus (\mathbf{10},\mathbf{1}) \oplus (\mathbf{1},\mathbf{2})$$

$$\mathcal{A}_\mu{}^M \;:\; \{\mathcal{A}_\mu{}^m, \mathcal{A}_{\mu\, m\,\alpha}, \mathcal{A}_{\mu\, kmn}, \mathcal{A}_{\mu\,\alpha}\}$$

$$\mathcal{B}_{\mu\nu\, M} \;:\; \{\mathcal{B}_{\mu\nu\, m}, \mathcal{B}_{\mu\nu}{}^{m\,\alpha}, \mathcal{B}_{\mu\nu\, mn}, \mathcal{B}_{\mu\nu}{}^{\alpha}\}$$

5+5 Kaluza-Klein decomposition

Schematically: $\{x^{\hat{\mu}}\} \to \{x^{\mu}, y^{m}\}$, $\hat{\mu} = 0..9$, $\mu = 0..4$, m=1..5

$$\hat{C}_{\hat{\mu}\hat{\nu}}{}^{\alpha} \longrightarrow \{C_{\mu\nu}{}^{\alpha},\ C_{\mu\, m}{}^{\alpha},\ C_{mn}{}^{\alpha}\}$$
$$\hat{C}_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}} \longrightarrow \{C_{\mu\nu\sigma\rho},\ C_{\mu\nu\rho\, m},\ C_{\mu\nu\, mn},\ C_{\mu\, nkl},\ C_{mnkl}\}$$

Remaining fields after EFT fields after dualization:

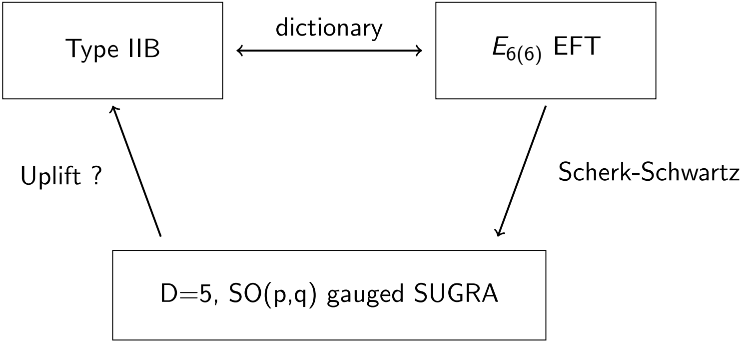
$$\{\Phi,\ m_{mn},\ b_{mn}{}^{\alpha},\ c_{mnkl},\ \mathcal{A}_{\mu\, m\alpha},\ \mathcal{A}_{\mu\, kmn}, \mathcal{B}_{\mu\nu\, mn},\ \mathcal{B}_{\mu\nu}{}^{\alpha}\}$$

Gauge transformation matching $\rightarrow$ IIB $\leftrightarrow$ $E_{6(6)}$ dictionary

$$A_{\mu}{}^{m} = \mathcal{A}_{\mu}{}^{m}, \quad C_{\mu\, m}{}^{\alpha} = -\epsilon^{\alpha\beta}\mathcal{A}_{\mu\, m\beta}, \quad C_{\mu\nu}{}^{\alpha} = \widetilde{\mathcal{B}}_{\mu\nu}{}^{\alpha},$$
$$C_{\mu\nu\, mn} = \frac{\sqrt{2}}{4}\widetilde{\mathcal{B}}_{\mu\nu\, mn}, \quad C_{\mu\, kmn} = \frac{\sqrt{2}}{4}\mathcal{A}_{\mu\, kmn}$$

$$
\begin{aligned}
C_{\mu kmn} &= \frac{\sqrt{2}}{4}\,\mathcal{Z}_{[ab]\,kmn}(y)\,A_{\mu}{}^{ab}\;, \\
C_{\mu\nu\,mn} &= \frac{\sqrt{2}}{4}\,\mathcal{K}_{[ab]}{}^{k}(y)\mathcal{Z}_{[cd]\,kmn}(y)\,A_{[\mu}{}^{ab}(x)A_{\nu]}{}^{cd}(x)\;, \\
C_{m\,\mu\nu\rho} &= -\frac{1}{32}\,\mathcal{K}_{[ab]\,m}(y)\left(2\sqrt{|\mathrm{g}|}\,\varepsilon_{\mu\nu\rho\sigma\tau}\,M_{ab,N}F^{\sigma\tau\,N}+\sqrt{2}\,\varepsilon_{abcdef}\,\Omega^{cdef}_{\mu\nu\rho}\right) \\
&\quad -\frac{1}{4}\sqrt{2}\,\mathcal{K}_{[ab]}{}^{k}(y)\mathcal{K}_{[cd]}{}^{l}(y)\mathcal{Z}_{[ef]\,mkl}(y)\left(A_{[\mu}{}^{ab}A_{\nu}{}^{cd}A_{\rho]}{}^{ef}\right)\;, \\
C_{\mu\nu\rho\sigma} &= -\frac{1}{16}\,\mathcal{Y}_{a}(y)\mathcal{Y}^{b}(y)\Big(\sqrt{|\mathrm{g}|}\,\varepsilon_{\mu\nu\rho\sigma\tau}D^{\tau}M_{bc,N}M^{N\,ca} \\
&\quad +2\sqrt{2}\,\varepsilon_{cdefgb}\,F_{[\mu\nu}{}^{cd}A_{\rho}{}^{ef}A_{\sigma]}{}^{ga}\Big) \\
&\quad +\frac{1}{4}\Big(\sqrt{2}\,\mathcal{K}^{k}_{[ab]}(y)\mathcal{K}^{l}_{[cd]}(y)\mathcal{K}^{n}_{[ef]}(y)\mathcal{Z}_{[gh]\,kln}(y) \\
&\quad -\mathcal{Y}_{h}(y)\mathcal{Y}^{j}(y)\,\varepsilon_{abcegj}\,\eta_{df}\Big)A_{[\mu}{}^{ab}A_{\nu}{}^{cd}A_{\rho}{}^{ef}A_{\sigma]}{}^{gh}+\Lambda_{\mu\nu\rho\sigma}(x)\;.
\end{aligned}
$$

Conclusion



Complete proof of Kaluza-Klein consistency of $AdS_5 \times S^5$:
Any solution of the D=5 maximal SO(p,q) (p+q=6) gauged supegravities lift to solutions of the type IIB field equations.